

## Sample Questions

Don't use calculators.

1. Let  $X$  be a 3-element set and  $Y$  be a 5-element set. If  $X$  and  $Y$  are disjoint, how many elements are there in their union  $X \cup Y$ ?
2. Suppose that each of four pieces of paper has one of the four compass points (N for north, S for south, E for east and W for west) written on it, with no two pieces showing the same letter. The four pieces of paper are put in a box.
  - (a) A researcher asks a volunteer to toss a coin or take a piece of paper from the box. How many possible outcomes are there?
  - (b) Now the volunteer can toss a coin or cast a die or reach into the box. How many possible outcomes are there?
3. The volunteer of Question 2 is now instructed to toss the coin and reach into the box.
  - (a) How many possible outcomes are there?
  - (b) Now the volunteer has to toss the coin and cast the die and reach into the box. How many possible outcomes are there?
4. The items in the box are now changed. The box now has five pieces of paper in it, with H (for head) on one, N (for north) on another, E (for east) on another, 2 on another and 5 on the last one.
  - (a) Once again the volunteer of Question 2 is instructed to toss the coin or reach into the box. How many possible outcomes are there?
  - (b) Now the volunteer has to toss the coin or cast the die or reach into the box. How many possible outcomes are there?
  - (c) If the volunteer has to toss the coin and cast the die and reach into the box. How many possible outcomes are there?
5.
  - (a) Evaluate:  $6!$
  - (b) Evaluate  ${}^6P_4$ .
  - (c) Evaluate  ${}^6C_4$ .
6.
  - (a) In how many ways can a set of 5 elements be arranged?
  - (b) In how many ways can an executive of 3 people be selected from an organisation of 8 people?
  - (c) In how many ways can an executive consisting of a president, a secretary and a treasurer be selected from an organisation of 8 people?
  - (d) The Koo-wee-rup Marbles Training Squad has 16 members, but only 14 can be selected for the big marbles game against Nar Nar Goon. In how many ways can the team of 14 be selected?
7.
  - (a) An accountant decides to hold a “new financial year” party. She will give a prize to any guest who has the same birthday as another guest. How many people will have to attend the party for her to be sure that a prize will be awarded?
  - (b) Have a guess at the answer to a related question: How many people would need to attend so that it is “more likely than not” that two people will have the same birthday? [Note: the answer to this question relies on probability theory, which comes up later in the course.]

- (c) A climate refugee from Kiribati has 6 pairs of ear muffs, with each pair being coloured differently from all the other pairs. On a cold day in Cairns he reaches into his darkened clothes dryer and starts pulling out ear muffs. How many must he pull out before it is certain that he will have two that are the same colour?
- (d) A maths enthusiast starts selecting integers from the set  $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$ . She will stop when two of the integers have the same square. What is the greatest number of integers she will need to select?